Flow over cube arrays of different packing densities

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Received 6 February 2006; received in revised form 9 November 2006; accepted 10 January 2007
Available online 23 March 2007

Abstract

Measurements by 120° x-wire anemometry over uniform urban-type surfaces of two different area densities were performed in a wind tunnel, together with direct measurements of the surface drag. The aerodynamic characteristics of each surface were determined and compared, the influence of area density and array geometry on these parameters was examined. Various approaches were discussed for the determination of the roughness length \((z_0)\) for a given surface. The surface shear stress (determined from form drag measurements by pressure tapping a roughness element or from the total surface drag determined by a floating drag balance) and the shear stress (determined from spatially averaged vertical profiles of Reynolds shear stress) were compared. The surface shear stress was found to be about 25% greater than the measured Reynolds shear stress in the inertial sub-layer over the surfaces. There was, however, no constant stress region and extrapolation of the shear stress profiles in the inertial sub-layer to the zero-plane displacement provided a much better estimate of the surface shear stress. The results did not support the argument put forward in the literature that the zero-plane displacement could be reliably predicted from the height of the centre of drag force. Finally, the accuracy of existing geometrical methods of determining the aerodynamic properties of rough surfaces was shown to be limited by the use of inappropriate wind tunnel data in their formulation.

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Keywords: Urban roughness; Drag force; Area density; Roughness length and wind tunnel

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1. Introduction

The estimation of surface characteristics is needed by many meteorological and wind engineering applications concerned with air pollution and environmental wind effects. The overall effect of the surface is often represented by the roughness length $z_0$ and/or the surface friction velocity $u^*$, both of which describe the roughness of the surface. In dispersion modelling, these are both important and essential input parameters. Accurate knowledge of the aerodynamic characteristics of rough surfaces is vital to the modelling and forecasting of the behaviour of urban wind and turbulence, as well as urban air quality.

As pointed out by Grimmond and Oke (1999), there are generally two different approaches to determine the aerodynamic characteristics of a given rough surface. One is a geometrical method, which determines the aerodynamic parameters from the surface geometry. This method is based on empirical relations derived from wind tunnel simulations over idealised surfaces. The second is a micrometeorological method, which uses direct field observation of wind and turbulence to deduce the roughness properties by assuming logarithmic wind profiles. The former is generally more widely used than the latter due to the high cost associated with operating experiments in the field. However, the “actual” or “true” surface characteristics can only be estimated from wind profile measurements taken over the surface (Petersen, 1997) and a reliable evaluation of $z_0$ and the zero-plane displacement $d$ requires the existence of a logarithmic layer in the flow over the rough surface (Bottema, 1997). Because of the height limitation of measurement towers, meteorological data are often obtained in the roughness sub-layer, which is likely to be inappropriate for overall parameterisation. Therefore, the understanding of boundary-layer meteorology leans heavily on experimental measurements in wind tunnels.

Grimmond and Oke (1999) also conducted sensitivity analysis on several morphometric methods of determining $z_0$ and $d$, and made comparison with values obtained from wind observations, but they still could not make a definitive judgement or provide recommendations concerning which methods should be used because of the lack of high quality measurements over urban roughness. After carefully screening the available data from the literature, they argued that there were few credible estimations of $z_0$ and almost none of $d$, therefore there was no credible standard against which to validate morphometric formulae.

In the early determinations of surface characteristics, the most common problem was a failure to include $d$ in the analysis—that is $d$ was often (inappropriately) assumed to be zero (Grimmond and Oke, 1999). For rough surfaces with very low obstacle density, where the zero-plane displacement may well be very small, this assumption probably has little effect on the determination of the roughness length. But for practical urban surfaces, the density is often relatively high. Spanton et al. (1996) surveyed some typical UK urban regions and found area densities as high as 50–60% in centrally situated commercial areas, while some industrial regions had area densities in the range of 20–40%. In such cases, the value of $d$ may be a significant fraction of the average building height, so the assumption of $d = 0$ could lead to significant error in estimating the value of $z_0$. For example, in our early study (Cheng and Castro, 2002a) of staggered cube arrays of 25% area coverage it was found that $d/h = 0.83$ and $z_0/h = 0.053$, where $h$ is the height of the roughness elements. If $d$ was forced to be zero, the log-law fitting process would lead to $z_0/h = 0.106$, which is 100% greater than it should be. The same behaviour was also observed by Bottema (1997).
It is our view that one of the main problems that causes the inaccuracy in determining $z_0$ is the fact that there is little information about the depth of the roughness sub-layer for a given surface and consequently inadequate wind speed data are often used to fit the log-law. Furthermore, measurements are often conducted at a location where the fetch is insufficient for the flow to become fully developed, so that a unique inertial sub-layer for the underlying surface does not exist. In that case, a log-law region is often inappropriately assumed in order to determine the surface properties (e.g. Macdonald et al., 2000), which leads to incorrect surface parameterisation. In addition, Counihan (1971) observed that the roughness length is a strong function of fetch, especially in the region before the flow becomes fully developed; this has been confirmed in our early work (Cheng and Castro, 2002a). It is generally accepted that the surface layer over roughness can be divided into the inertial sub-layer and roughness sub-layer. At the top of the roughness sub-layer, turbulent mixing smears individual wakes sufficiently to cause the flow to become independent of horizontal position and the flow “sees” the rough surface as uniformly rough and its detailed structure is unimportant. This height represents the minimum elevation above a rough surface at which the vertical profiles at an individual location are representative of the flow over the roughness.

The depth of the roughness sub-layer is often thought to be about $2–5h$, depending on the geometrical arrangement of the elements (Raupach et al., 1991), but the precise depth is unknown over most surfaces and no systematic studies are available in the literature. Information on the depth of the roughness sub-layer is very limited and confusing. O’Loughlin and Annambhotla (1969) examined water channel flow over cube roughened boundaries of low roughness concentration and suggested that the outer limit of the roughness sub-layer occurs at $z_r = 2h$. Mulhearn and Finnigan (1978), relying on wind tunnel experiments over random, rough gravel surfaces, suggested that $z_r = 2\bar{D}$ where $\bar{D}$ is the average spacing between the roughness elements. Raupach et al. (1980) studied regular cylinder arrays with five different densities and suggested that $z_r = h + 1.5B$ (where $B$ is the inter-element spacing), without indicating the influence of the area density on the depth of the roughness sub-layer. Our recent study (Cheng and Castro, 2002a) showed that the depth of the roughness sub-layer, which is essentially independent of fetch, is $1.8h$ for regular cube arrays and $2.5h$ for a particular random height surface, even though all the surfaces have the same area density of 25%. It is $4h$ for uniform, two dimensional, square bar rib-type roughness and $5h$ for two dimensional, vertical flat plates (Cheng and Castro, 2002b), where $h$ is the mean height of the roughness elements. These few expressions for the depth of the roughness sub-layer available in the literature probably afford a good indication of our limited current understanding of its characteristics.

It is well accepted that there are three possible flow regimes over regular urban roughness (Oke, 1987). In the isolated flow regime, the wake and the separation bubble behind each obstacle is fully developed, with re-attachment occurring before the next element. The bulk of the flow is forcibly displaced up and over the obstacle, which causes acceleration or a jet, but once over it is able to expand again and decelerates accordingly. This flow region, disturbed because of the presence of the obstacle, is called the displacement zone. With increasing density, the roughness elements become close enough so that the wake behind an obstacle starts to interfere with that of the downstream obstacle, leading to a complicated flow pattern. If the density increases further, the flow begins to skim over the elements. It has been reported that the maximum value of roughness length is obtained with the onset of wake interference flow (Jia et al., 1998).
The depth of the roughness sub-layer is probably associated with the height of the displacement zone. The size and shape of the wake shed by a building depends on its width and depth as well as its height and distance to surrounding structures. It seems likely that the depth of the roughness sub-layer depends on the geometry of the rough surface, such as the shape and size of the obstacles, their layout and the consequent area density.

Some studies (Raupach et al., 1991; Macdonald et al., 1998; Grimmond and Oke, 1999) have shown that the roughness length, expressed as a fraction of building height, depends on frontal area density and/or plan area density. The roughness length increases with increasing area density, until a point comes where adding new roughness elements merely serves to reduce the effective drag of those already present, due to mutual sheltering. At the extreme density of 100% the elements are so closely packed that a new smooth surface is formed. This behaviour produces a peak in $z_0$ at some intermediate density. Similarly, with increasing density, the zero-plane displacement $d$ starts to move upwards and eventually the packing becomes so dense that eddies have difficulty penetrating the inter-element space and the flow skims, so the level of the element roof becomes the new displacement height for the mean flow. Bottema (1997) presented an analytical model for evaluation of $z_0$ and $d$ for regular building groups, and suggested that apart from the frontal density, the stream-wise building length and the building layout pattern were also important parameters.

There also exists a great deal of confusion about the zero-plane displacement $d$ in the literature. The roughness length $z_0$ is regarded as the basic measure of the degree of surface roughness and by far the more significant parameter than $d$. In atmospheric measurements, one typically does not know $u^*$, $z_0$ and $d$, so all three unknowns have to be found from the log-law fitting of a velocity profile measured at a few heights. To reduce the uncertainty in this fitting process, $d$ is often assumed or approximated by the investigators, as in the work of Macdonald et al. (1998, 2000), which in turn contributes to the variability in evaluations of $z_0$, even for a given type of roughness. In more advanced studies, which are usually wind tunnel simulations, $u^*$ is independently determined from the measured vertical profiles of the Reynolds shear stress. Even so, there still exists uncertainty in choosing which region of the velocity profile to fit with the log-law.

In all the cases mentioned above $d$ is no more than a fitting parameter. However, Jackson (1981) argued that the displacement height in the log-law should be regarded as the level at which the mean drag acts on the surface. He further suggested that $d$ is determined by the distribution of the force on the surface, whereas the roughness length is determined by the magnitude of this force. Jackson’s method is intuitively appealing, not only because $d$ is determined independently of the log-law fitting, but also because he gives $d$ a physical meaning. Most recently Kastner-Klein and Rotach (2004) performed measurements at a few locations in the centreline of a long street over a complicated urban model. Because of the long and deep urban street canyon nature of their model, they observed negligible momentum flux at the lower section of the street and pronounced maxima above roof level in their shear stress profiles. They then introduced a shear stress displacement height $d_s$ and a length scale $z_s$, which is the height at which the maximum shear stress occurs. Based on their limited measurements, they formulated mathematical expressions for $d_s$, $z_s$ (as a function of plan area density) and shear stress profile (as a function of height) by quasi-physical arguments and curve fitting. By defining the zero-plane displacement $d$ as the level of mean momentum absorption (Jackson, 1981) and relating their roughness length to $(z_s-d_s)$, they pointed out that both $z_0$ and $d$ can be
expressed by linear correlations as a function of distance between the level of shear stress peak and the shear stress displacement height ($z_s - d_s$). It is remarkable that by establishing the variation of the plan area density over a small area of complicated urban model they could then derive the roughness length for each individual vertical velocity profile; it is unlikely though that this would always be a reliable approach.

In addition, a recent review by Jiménez (2004) has reiterated the importance of the ratio $\delta/h$ in understanding turbulent flow over rough walls ($\delta$ and $h$ are the boundary-layer thickness and the height of the roughness elements, respectively). Most of his review discussed the case of large $\delta/h$ but he pointed out that, for flows with small $\delta/h$, few of the mechanisms of normal classical wall turbulence remain and the influence of the roughness expands well beyond the near-wall region, if not across the whole boundary layer. For many practical flows in urban areas, the validity of the log-profile is questionable because the values of $\delta/h$ may well not be large. However, it is necessary to investigate flows in such situations in order to understand how pollutants are dispersed and transported in urban streets, and this paper is specifically focused on these cases.

The development of characteristic surface flow properties and length scales was investigated in a previous study, (Cheng and Castro, 2002a), and some of the conclusions of that work are worth reiterating here. The following observations refer to a staggered, regular array of 25% area density. The depth of the roughness sub-layer, after a short development fetch, was found to be about $1.8h$, independent of fetch and surface geometry. The depth of the inertial sub-layer also became constant after a short development fetch, though it decreased relative to the boundary-layer depth, becoming about 0.25$\delta$ in the far-field, beyond about a fetch equivalent to $\delta/h = 12$. The roughness length, expressed as $z_0/h$, increased with fetch, becoming constant at about $\delta/h = 7$. Finally, the friction velocity, expressed as $u^*/U_r$, decreased with increasing fetch over the range studied, equivalent to $\delta/h = 4$–12. The boundary layer did not become genuinely self-preserving over this fetch. The study concluded that the derivation of the friction velocity by conventional methods, even using spatially averaged profiles, was not reliable and recommended more direct means of measurement (as investigated in the current work).

The aim of the present study is to investigate the effects of surface geometry on the aerodynamic characteristics of urban-like roughness, using direct measurements of surface stress in the analysis. This is a natural continuation of work described in Cheng and Castro (2002a). In the present paper, we will describe wind tunnel experimental results over uniform cube arrays of two different area densities to examine the influence of area density on surface characteristics at a blockage ratio ($\delta/h$) of 6–7. Surface shear stress, determined from both form drag measurements (using a pressure-tapped roughness element) and total surface drag (obtained by a floating drag balance), are compared with the Reynolds shear stress in the inertial sub-layer as measured by 120° x-wire anemometry. We will compare the values of roughness length using the same range of the wind speed profile for a given surface with different $u^*$, and test whether the height at which the mean drag force acts is the same as the zero-plane displacement.

2. Experimental arrangement

The experiments were conducted in the EnFlo “A” wind tunnel, which has a test section 4.5 m long 0.6 m high and 0.9 m wide. The free stream velocity in the wind tunnel was
measured by a Pitot static tube. \((x, y, z)\) are the stream-wise, lateral and vertical
coordinates, with the plane \(z = 0\) being the top surface of the baseboard on which the
roughness elements were located. Time and space averaged mean and fluctuating velocities
will be denoted as \((U, V, W)\) and \((u', v', w')\), respectively. Over-bars denote time averages
and angle brackets denote spatial averages.

In this study, two area coverage densities (the ratio of the plan area occupied by the
roughness elements to the total surface area) of 25% and 6.25% were used with four rough
surfaces, with the cubes in staggered or aligned patterns as shown in Fig. 1 (when installed
so that the wind flows from left to right the cubes are aligned, when from top to bottom
they are staggered). The region surrounded by the dashed line defines the module area
occupied by a cube and “+” denotes the locations of the vertical profiles. There are, of
course, many ways of defining an appropriate module but this has no bearing on the
outcome of the spatial averaging. The roughness elements for all the surfaces were sharp
edged wooden cubes of side 20 mm. In order to prevent interference of the boundary layer
from the upstream contraction of the wind tunnel, the rough surfaces were raised by
60 mm above the tunnel floor and a section of aluminium flat plate with rounded leading
edge was positioned at the front of the rough surface. In each case the roughness covered
the entire length of the tunnel floor. All the experiments described here were performed on
the tunnel centreline and about 3 m \((x/h = 150)\) downstream of the roughness leading edge.

The vertical profiles of mean flow and turbulence were measured by 120° crossed hot-
wire anemometry, with corrections for high turbulence intensity (Tutu and Chevray, 1975),
at a free stream velocity of 10 m/s. Form drag measurements were performed at free stream
velocities of 4–10 m/s by replacing one wooden cube with a brass one that was fitted with
42 pressure tappings, 21 in the front face and 21 in the rear face. These were positioned as
shown in Fig. 2, so that pressures at 42 locations on the front and rear faces were obtained
by rotating the brass cube through 180°. Pressures were measured with a micro-manometer

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Fig. 1. Schematic diagram of the rough surfaces investigated: (a) 6.25% cube arrays and (b) 25% cube arrays.
and a scani-valve. The static pressure from a Pitot-static tube, which was located in the free-stream above the measurement position, was used as the reference pressure. A sampling frequency of 200 Hz and a sampling time of about one minute at each position were generally used for both $x$-wire and form drag measurements. It is estimated that the standard errors associated with this finite sampling time are well below 1% of the mean velocity.

The total surface drag was also measured by a floating drag balance, which is briefly described as follows. Roughness elements (in total, 18 for the 25% surfaces and 7 for the 6.25% surfaces) were glued in exactly the same pattern as in the roughness arrays on a foam sheet to form a "raft", as shown in Fig. 3. An oil bath, in which the raft was placed, was located on the tunnel centre-line at about 3 m downstream of the leading edge of the roughness array; its plan area was larger than the raft, so that there was a 20 mm gap between the sides of the raft and the edges of the bath. This reasonably large gap was necessary to avoid surface tension causing the raft to be attracted to the bath edges. The buoyancy of the raft was designed so that the top of the foam surface was just above the oil level, and the bath was filled so that the height of the cubes on the raft matched the surrounding array. EP80 gear oil was chosen as the working fluid in the bath as its high viscosity was useful for damping oscillation of the raft during the experiment. The viscosity

Fig. 2. Pressure tapping distribution on the brass cube (solid and open symbols denote opposite faces).
was also important in preventing the oil surface around the raft becoming too perturbed by the moving air above. By floating the raft, air leakage was prevented and a system having theoretically zero internal friction was produced.

The raft was tethered to a vertical tube (acting as a cantilever spring) that was held by the overhead traversing system, whose longitudinal position was adjusted to maintain the raft in the correct position relative to the surrounding array. This arrangement enabled the aerodynamic drag force acting on the cube and raft surfaces to be obtained by measuring the cantilever deflection (via the change in traverse position) over a range of tunnel speeds from 4 to 12 m/s. The stiffness of the cantilever spring was calibrated before the experiments and compensation was made for the force acting on the spring surface itself.

3. Form drag measurements

The drag force $D$ on an individual cube was determined by integrating over the frontal area the pressure difference between the front and back faces of the cube at corresponding positions:

$$D = \int_A (p_f - p_b) \, dA,$$

(1)
where \( p_f \) and \( p_b \) are the pressures on the front and back faces, respectively, and \( A \) is the frontal area of the roughness element. A first-order integration scheme was used, assuming constant pressure across the elements of area shown in Fig. 2. Further analysis using a higher-order integrating scheme confirmed that this technique was adequate.

Based on the free stream velocity, the drag coefficient of an individual cube is expressed as

\[
C_d^r = \frac{2D}{\rho U_f^2 A}.
\]  

(2)

Neglecting viscous drag, the surface shear stress over the module area \( A_c \) is given by

\[
\tau_p = \frac{D}{A_c}
\]  

(3)

and the friction velocity of the surface is defined as

\[
u^*(p) = \sqrt{\frac{\tau_p}{\rho}}.
\]  

(4)

From the above equations the relationship between the drag coefficient of an individual roughness element and the friction velocity of the rough surface can be established as

\[
\frac{u^*(p)}{U_f} = \left( \frac{C_d^r \lambda_f}{2} \right)^{0.5},
\]  

(5)

where \( U_f \) is the free stream velocity, \( \tau_p \) is the shear stress due to the form drag, \( u^*(p) \) is the friction velocity determined from the form drag, \( \rho \) is the air density and \( \lambda_f = A/A_c \). \( C_d^r \) is the cube drag coefficient based on the free stream velocity. If the drag coefficient is based on the friction velocity, the expression \( C_d^* = \frac{2}{\lambda_f} \) can be derived through re-arrangement of Eqs. (2)–(4). This drag coefficient is only a function of the frontal area density, and is therefore 32 and 8 for the rough surfaces with area density of 6.25% and 25%, respectively.

The results for \( C_d^r \) and \( u^*(p)/U_f \) versus free stream velocity are displayed in Fig. 4(a) and (b) respectively. Both the drag coefficient and the friction velocity are only a weak function of free stream velocity (or Reynolds number) in the conditions investigated. These results also showed that the staggered arrays produced higher resistance to the flow than the aligned cube arrays (this is more evident at lower than higher area coverage), as illustrated in Fig. 4(c). The drag coefficient of an individual element in a cube array is significantly smaller than that of an isolated cube in a turbulent boundary layer (typical values can be found in Akins and Peterka (1977)) due to the sheltering effects in the cube arrays.

With increasing surface area coverage density, the drag force experienced by individual roughness elements decreases, as does the drag coefficient, which is due to the mutual sheltering effects at higher packing density. However, comparison of the \( u^*(p)/U_f \) data for the four rough surfaces shows that the ratio of the friction velocities for packing density of 6.25% and 25% is about 1 for the staggered pattern and 0.94 for the aligned pattern and almost independent of the free stream velocity, as shown in Fig. 4(d). For the staggered arrays, the surface stresses for both surfaces are almost identical, even though the roughness element arrangement in one is four times denser than the other, and the flows over the 6.25% and 25% surfaces are characterised by different flow regimes, as discussed
later in this section. This is purely coincidental. The roughness length passes through a maximum as the packing density increases from 0 to 1 and two surfaces happen to have been chosen that straddle that maximum and give almost identical surface stresses.

By integrating the pressure distribution on the roughness element, the height of the centre of drag $H_c$ can be determined from the following expression:

$$
Z_A z(p_f/C_0 p_b) \,dA = DH_c, \tag{6}
$$

where $H_c$ is the height at which the mean drag force acts on the roughness elements. The results normalised by cube height for all four surfaces are plotted against free stream velocity in Fig. 5. It is evident that the height of the centre of drag is independent of both
the magnitude and the direction of the free stream velocity and is only a function of area coverage density in the conditions investigated, which implies that in these cases \( H_c \) is determined by the surface geometry rather than the flow.

As stated in Section 2, the pressure distributions on the roughness elements were obtained for all four surfaces at free stream velocities of 4–10 m/s. The results are plotted non-dimensionally in the form of contours of constant local pressure difference coefficient \( C_p \), defined as

\[
C_p = \frac{(p_f - p_b)}{\frac{1}{2} \rho U_r^2}.
\]  

(7)

As for the drag coefficient shown in Fig. 5(a), the influence of the free stream velocity on the pressure coefficients was insignificant; for each surface the \( C_p \) distributions at different free stream velocities remained similar and only those at 10 m/s are displayed in Fig. 6. Comparison of the corresponding cube arrays with different area densities shows that the pressure coefficients on the individual cubes for 6.25% surfaces have much higher values than those of the 25% surfaces, which is undoubtedly due to the greater shelter effects at higher density. For the 25% surface, whether the roughness elements are arranged in the staggered or aligned pattern, the range of \( C_p \) values is quite similar; this is probably because the packing density is so high that the flow above cannot significantly penetrate into the urban canopy layer regardless of the wind direction. For the 6.25% surfaces, the distribution of \( C_p \) is very similar to the results of the pressure distribution on an isolated cube immersed in a boundary-layer flow; see Castro and Robins (1977) and Hunt (1982).

The largest pressure difference was observed in the region around \( z = 16 \text{ mm} \) for the 6.25% surfaces, whereas it is at the very top of the cube for the 25% surfaces. It seems clear that the location of the high-pressure area on the cube moves upward as the packing density increases until it reaches the top of the cube when the flow begins to skim the
roughness elements. It is possible that, for the 20 mm cube arrays, the flow has started to skim the roughness elements at an area density well below 25%. These results also confirmed that the staggered cube arrays experienced higher drag force than the aligned pattern for the 6.25% surfaces (similarly for the 25% surfaces, but to a less pronounced
degree), which is consistent with the previous discussions of drag coefficient associated with the data shown in Fig. 4.

4. Vertical profiles results

4.1. The roughness sub-layer

For each of the four rough surfaces investigated, 16 vertical profiles of mean flow and turbulence properties (uniformly distributed over a module, as shown in Fig. 1), were taken for a free stream velocity of 10 m/s, as mentioned previously. The vertical profiles are displayed in Figs. 7–10, for C20S-25%, C20A-25%, C20S-6.25% and C20A-6.25% respectively; here C, S and A stand, respectively, for cube, staggered and aligned. Comparison of these data among the four roughness conditions shows that C20A-6.25% is an exceptional case. It is evident that the flow feels the presence of the surface details at least up to a height around two-thirds of the boundary-layer thickness. The roughness elements were so sparsely spaced in a span-wise sense that the flow in the wide channels between the cubes remains almost unaffected by the flow over the rows of cubes.

Fig. 7. Vertical profiles for C20S-25%: (a) $U/U_r$; (b) $u'^2/U_r^2$; (c) $w'^2/U_r^2$; and (d) $uw'/U_r^2$. 
This makes parameterisation from one single vertical profile impossible for the surface. It is noteworthy that the ratios of $\delta/h$ for all the four surfaces studied are about 6~7; the height of the roughness element is a significantly large fraction of the total boundary-layer depth. The results for C20A-6.25% roughness have certainly confirmed the statement made by Jiménez (2004) that the effects of the roughness extend across the boundary-layer depth for flows with a low value of $\delta/h$. Whether the flow over such a surface will develop into a state where the effects of the roughness elements vanish at a certain height and the logarithmic layer is found at great fetch, as claimed by Jiménez (2004), is beyond the scope of this study and remains to be answered. However, these results have demonstrated that the blockage ratio of $\delta/h$ is not the only parameter that controls whether an inertial sub-layer exists. They seem to indicate that the layout pattern (or the inter-element distance in the case of C20A-6.25%) has an important role to play in determining the depth of the roughness sub-layer. The vertical profiles of spatially averaged shear stress over a modular unit for all the surfaces studied are displayed in Fig. 11. It is interesting to note that the profile shapes of the spatially averaged shear stress near the surface are different for staggered and aligned arrays. This must reflect the differing degrees of shelter (or exposure) in these two types of array.
The roughness sub-layer is generally defined as the region where the flow is strongly influenced by individual roughness elements. In this study, the upper limit of the roughness sub-layer is defined qualitatively as the height where the influence of the geometry of the surface disappears and all the vertical profiles within a module converge. Because of the low ratio of $d/h$ in the flows involved here, a logarithmic layer (clearly seen in a rough wall boundary layer with larger $d/h$, such as $d/h > 80$ (Jiménez, 2004) and, of course, over a smooth wall) may not exist at all. As an attempt to find a way to describe the flow over practical urban surfaces, the depth of the inertial sub-layer is tentatively defined by the region above the roughness sub-layer within which the vertical variation in spatially averaged shear stress is confined to 5%. These definitions are somewhat subjective but alternative definitions, which are examined and discussed in Section 4.4, do not affect the main conclusions. From Figs. 7–10, it is clear that in the inertial sub-layer the variation with profile location within the module is relatively small compared to that in the roughness sub-layer. As pointed out by Feddersen et al. (2003), there exist two definitions for the inertial sub-layer, one is the quasi-constant flux layer used in our early work (Cheng and Castro, 2002a) and the other is the region over which the wind profile can be fitted by the log-law. In this paper, we will examine whether these two definitions lead to similar
surface parameters for a given roughness. The thickness of the boundary layer at the
measurement location was defined as the height at which the mean velocity was 99% of the
free stream velocity. The depths of each layer for each surface are listed in Table 1 and also
indicated in Figs. 7–10. Apart from the exceptional case of C20A-6.25%, these results
indicate that the depth of the roughness sub-layer is essentially independent of the area
coverage density and the wind direction for the conditions investigated here.

4.2. Surface shear stress

Raupach (1992) re-analysed the data given by Marshall (1971) and deduced that the
form drag is the dominant surface force for rough surfaces with an area density of more
than 3%. Most recently, Coceal et al. (2006) performed direct numerical simulation (DNS)
for flows over 25% cube arrays (the same surfaces as used in this study) and analysed the
vertical profiles of the stress tensor, which consist of viscous, Reynolds and dispersive
stresses. They found that the viscous term is negligible except on the top surface of the
cubes, where its maximum contribution is only about 3% of the total stress. Furthermore,
Snyder and Castro (2002) indicated that viscous effects are insignificant once the surface is
fully aerodynamically rough, which, for sharp-edged obstacles, requires the roughness Reynolds number \( Re_c = u^*z_0/v > 1 \). For the roughness and experimental conditions investigated here, \( Re_c \) is at least 15.

For each roughness, the surface stress \( \tau_p \), derived from form drag measurements, was used as the baseline and compared with the Reynolds shear stress \( \tau_m \) (here \( \tau_m = \rho(-\overline{u'w'}) \)), deduced from the spatially averaged shear stress profiles within the inertial sub-layer over the surface at the same flow conditions. The shear stress ratios for all four surfaces are listed in Table 1; the non-dimensional values of \( u^*(p)/U_r \) derived from \( \tau_p \) are also included in Figs. 7–10. These data clearly demonstrate that the surface shear stress is in general

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**Table 1**

Summary of the various layers over the rough surfaces, evaluated at \( U_r = 10 \text{ m/s} \)

<table>
<thead>
<tr>
<th>Layer Description</th>
<th>C20S-25%</th>
<th>C20A-25%</th>
<th>C20S-6.25%</th>
<th>C20A-6.25%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Boundary layer depth ( \delta )</td>
<td>6.7h</td>
<td>6.9h</td>
<td>6.8h</td>
<td>6.2h</td>
</tr>
<tr>
<td>2 Top of roughness sub-layer</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Depth of inertial sub-layer (5% variation in ( \overline{u'w'} ))</td>
<td>1.75~2.15h</td>
<td>1.8~2.4h</td>
<td>1.8~2.2h</td>
<td>~4h</td>
</tr>
<tr>
<td>4 ( Z_{top of inertial sub-layer/\delta} )</td>
<td>32%</td>
<td>33%</td>
<td>33%</td>
<td></td>
</tr>
<tr>
<td>5 Vertical region fitted by Log-law</td>
<td>1.2~2.75h</td>
<td>1.2~3h</td>
<td>1.2~3h</td>
<td>1.2~2.55h</td>
</tr>
<tr>
<td>6 Equivalent variation in ( \overline{u'w'} % ) for log-law region</td>
<td>14%</td>
<td>10%</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>7 ( u^*(p)/U_r )</td>
<td>0.0718</td>
<td>0.0682</td>
<td>0.0728</td>
<td>0.0642</td>
</tr>
<tr>
<td>8 ( \tau_p/\tau_m )</td>
<td>1.26</td>
<td>1.22</td>
<td>1.32</td>
<td></td>
</tr>
<tr>
<td>9 ( \tau_p/\tau_d )</td>
<td>1.03</td>
<td>0.9</td>
<td>1.12</td>
<td>0.97</td>
</tr>
</tbody>
</table>

---

**Fig. 11.** Vertical profiles of spatially averaged shear stress over a modular unit for the four surfaces investigated.
about 25% larger than the Reynolds shear stress directly measured in the inertial sub-layer. A similar phenomenon was also observed by Iyengar and Farell (2001) with a smaller difference of 18% in their experiments, and they suspected that the discrepancy was due to cross wire errors in the near wall region where the turbulent intensity was high. It is worth mentioning that they used 90° x-wires in their study. However, in our early work (Cheng and Castro, 2002a), the adequacy of the 120° x-wire data was checked and confirmed by comparison with laser Doppler anemometry data. Consequently, it is believed that the discrepancy between the surface shear stress \( \tau_p \) and the inertial sub-layer Reynolds shear stress \( \tau_m \) cannot be attributed to the measurement technique used in this investigation. Nonetheless, in all cases it is significant that plausible extrapolations of \(-\overline{u'w'}\) from the inertial sub-layer (in which it is not, in fact, constant) to the point \( (z = d) \) do yield values relatively consistent with the surface stresses implied by the pressure measurements. In any case, deriving a friction velocity from shear stress measurements aloft, which is common practice in wind tunnel simulation of the atmospheric boundary layer, would significantly underestimate the surface friction velocity.

The longitudinal pressure gradient, \( d \bar{P}/dx \) (see Eq. (9) in Section 4.4), resulting from boundary-layer growth over the rough lower wall and the other three working section surfaces was estimated. This was found to be consistent with the observed shear stress gradients near the wall. The inevitable consequence is that estimates of the friction velocity from measurements above the roughness sub-layer should take the gradient into account in extrapolating down to the zero-plane level. (Table 1 lists a boundary-layer depth of about 6\( h \), which was about 20% of the working section depth; the growth rate was estimated as \( d\delta/dx \approx 0.03 \).) This argument implies that shear stress profiles in many wind tunnel experiments will be affected by small axial pressure gradients unless either the working section is very deep indeed or the working section roof can be adjusted to keep \( U_r \) constant.

It is important to note that Reynolds et al. (2007) have recently shown that significant span-wise variations can occur in boundary layers developing over regular arrays. However, in the present work the surface shear stress, \( \tau_p \), has been derived from the pressure distribution around a single cube, rather than as an average over many cubes. Such an average can be obtained with a drag balance.

As mentioned in Section 2, a floating drag balance was also used to obtain the drag force acting on a number of cubes and the raft surface on which they were mounted. An average drag coefficient was obtained by assuming that form drag dominated and dividing the measured drag force by the number of cubes on the raft. The results, also plotted in Fig. 4(a), show generally good agreement with the pressure drag measurements for the four surfaces over the free stream velocity range of 4–12 m/s. This gives credibility to the floating drag balance method since consistent drag coefficient values were obtained for a speed range over which the drag force changed by a factor of 9. More recent studies (not yet published) on pressure measurements in the cube arrays have shown that the drag measured on a single cube has a span-wise variability of around \( \pm 10\% \) due to the span-wise variation of velocity in the boundary layer (Reynolds et al., 2007). This could readily account for the differences between the measurement techniques for the staggered 6.25% case and suggests that the very close agreement in the other three cases may be somewhat fortuitous. It should be noted that the lateral dimensions of the raft approximately correspond with a single wavelength of the spanwise variation observed in the velocity profiles.

The surface shear stress \( \tau_p \) was calculated by dividing the drag measured by the floating balance by the total modular surface area associated with the number of cubes on the raft.
A small error, that the DNS simulations of Coceal et al. (2006) suggest is likely to be negligible (at least, for the 25% density arrays), is associated with this as the raft area was only 70% of the total modular area, due to the 20 mm gap surrounding the raft. The measurements were compared with $\tau_p$ at the same flow condition and the ratio of $\tau_p/\tau_d$ for a free stream velocity of 10 m/s is included in the final row of Table 1. Given the $\pm 10\%$ uncertainty in the cube pressure drag measurements associated with the $\pm 5\%$ span-wise variation in velocity in the boundary layer, the ratios are as close to unity as could reasonably be expected.

4.3. Log-law fitting procedure

In the log-law fitting process described here, the friction velocity is pre-determined from either form drag measurements $u^*(p)$ or the shear stress profile $u^*(m)$. If there are $n$ data points in the region that is selected to fit the logarithmic wind profile, the sum of the least-square error for the selected region can be obtained from the difference between the measured velocity and the calculated one from the usual logarithmic law by the following expression:

$$E = \sum_{i=1}^{n} \left[ U_i - \frac{u^*}{k} \ln \left( \frac{z_i - d}{z_0} \right) \right]^2,$$

where $U_i$ is the measured mean velocity at height of $z_i$ in the selected region. Both the zero-plane displacement $d$ and the roughness length $z_0$ are unknowns, and assigned different values. For each pair of assigned $d$ and $z_0$ the total error $E$ was evaluated and the fitting was visually inspected. The values of $d$ and $z_0$, which gave a minimum error, are the required solution of the log-law fitting process. In this procedure $d$ was constrained to be less than or equal to the height of the roughness elements.

However, when all three unknown roughness parameters ($u^*$, $d$ and $z_0$) are all derived in this way from the mean velocity profiles, the final results obtained from the least-square fitting process are sensitive to the initial values assigned to the unknowns. Therefore, as has been pointed out before, this procedure is somewhat ill-conditioned and caution should be exercised when using such a method to determine the surface parameters from velocity measurements at a few heights, whether within wind tunnel flows or in the atmospheric boundary layer.

4.4. Discussions on parameterisation

For each surface, including C20A-6.25%, the vertical velocity profiles over a module were spatially averaged. The log-law was then fitted from the lowest measurement point, which was 4 mm above the cube top, through the roughness sub-layer and the inertial sub-layer, and into the flow above as far as was possible, provided that the fitting remained visually good (see Fig. 12), using $u^*(p)$ as the surface friction velocity. The upper limit of the region, which was well fitted by a log-law, is also shown in Figs. 7–10. The fitting parameters ($z_0$ and $d$) are listed in the first row of Table 2 and are used as the baseline when compared with $z_0$ and $d$ from other approaches.

The depth of the log-law region is different for different roughness conditions and is listed in the 5th row of Table 1. The equivalent variation in the spatially averaged shear
stress for this region is listed in the 6th row of the table. For the 25% surfaces, the log-law region would be consistent with the inertial sub-layer defined by the quasi-constant shear stress layer if the latter was based on a definition allowing a 10% variation in $\frac{\mu}{C_0 u_0}w_0$. However, for the 6.25% surfaces, it would require a 20% variation in $\frac{\mu}{C_0 u_0}w_0$ to unify the two approaches indicated by Feddersen et al. (2003). No single definition of the extent of the sub-layer will apply to all the cases examined. Clearly, the shear stress is not constant with height within the log-law region and to explain why, we examine the equations of motion.

Following Finnigan (2000) and Coceal and Belcher (2004), for ideal steady 2D flow over a rough wall, the time and space averaged momentum equation in the stream-wise direction is

$$\rho \left( U \frac{\partial U}{\partial x} + W \frac{\partial U}{\partial z} \right) = - \frac{dP}{dx} - \rho \left( \frac{\partial (u'^2)}{\partial x} + \frac{\partial (u'u'w')}{\partial z} \right) - D - \rho \left( \frac{\partial (\tilde{u}^2)}{\partial x} + \frac{\partial (\tilde{u}\tilde{w})}{\partial z} \right).$$

(9)
There are new terms on the right-hand side: $\left\langle \overline{u^2} \right\rangle$ and $\left\langle \overline{uw} \right\rangle$ are the spatially averaged Reynolds stresses, which represent spatially averaged momentum transport due to turbulent velocity fluctuations; $\left\langle \overline{u^2} \right\rangle$ and $\left\langle \overline{uw} \right\rangle$ are dispersive stresses, due to momentum transport by the spatial deviations from the time-averaged wind; $D$ is the drag force due to individual roughness elements (and is thus only non-zero within the canopy region). Experimental evidence has shown that, compared with the usual Reynolds stresses, the dispersive stresses are negligible in the region above the top of the canopy, but may be significant near the bottom of the canopy, though both stresses tend to be small in the canopy layer where the drag force is important. For fully developed flow where quantities do not change with $x$, under zero pressure gradient conditions, the above equation then reduces to

$$0 = -\rho \frac{\partial \left\langle \overline{uw} \right\rangle}{\partial z} - D - \rho \frac{\partial \left\langle \overline{uw} \right\rangle}{\partial z}$$

since $\overline{W} = 0$ in such circumstances. The convective and dispersive stress terms can be neglected in the log-law layer and $D = 0$ so that, as in canonical boundary layers, Eq. (10)

### Table 2
Surface properties calculated using $u^*(p)$ (except in the final two rows)

<table>
<thead>
<tr>
<th>Fitting region</th>
<th>C20S-25%</th>
<th>C20A-25%</th>
<th>C20S-6.25%</th>
<th>C20A-6.25%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-law region</td>
<td>$z_0/h$</td>
<td>0.049</td>
<td>0.044</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>$d/h$</td>
<td>0.858</td>
<td>0.853</td>
<td>0.558</td>
</tr>
<tr>
<td>RS and IS (5% variation in $-\overline{uw}$)</td>
<td>$z_0/h$</td>
<td>0.052</td>
<td>0.046</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>$d/h$</td>
<td>0.822</td>
<td>0.823</td>
<td>0.504</td>
</tr>
<tr>
<td>RS and IS (10% variation in $-\overline{uw}$)</td>
<td>$z_0/h$</td>
<td>0.050</td>
<td>0.044</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>$d/h$</td>
<td>0.845</td>
<td>0.853</td>
<td>0.504</td>
</tr>
<tr>
<td>RS and IS (20% variation in $-\overline{uw}$)</td>
<td>$z_0/h$</td>
<td></td>
<td></td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>$d/h$</td>
<td></td>
<td></td>
<td>0.558</td>
</tr>
<tr>
<td>IS (5% variation in $-\overline{uw}$)</td>
<td>$z_0/h$</td>
<td>0.045</td>
<td>0.039</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>$d/h$</td>
<td>0.964</td>
<td>1.000</td>
<td>0.618</td>
</tr>
<tr>
<td>IS (10% variation in $-\overline{uw}$)</td>
<td>$z_0/h$</td>
<td>0.043</td>
<td>0.038</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>$d/h$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.623</td>
</tr>
<tr>
<td>IS (20% variation in $-\overline{uw}$)</td>
<td>$z_0/h$</td>
<td></td>
<td></td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>$d/h$</td>
<td></td>
<td></td>
<td>0.719</td>
</tr>
<tr>
<td>Log-law region with $d = H_c$</td>
<td>$z_0/h$</td>
<td>0.063</td>
<td>0.056</td>
<td>0.046</td>
</tr>
<tr>
<td></td>
<td>$h_c/h$</td>
<td>0.612</td>
<td>0.616</td>
<td>0.526</td>
</tr>
<tr>
<td>Low-law region using $u^*(m)$</td>
<td>$z_0/h$</td>
<td>0.031</td>
<td>0.029</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>$d/h$</td>
<td>0.932</td>
<td>0.916</td>
<td>0.704</td>
</tr>
<tr>
<td>Low-law region with $d = H_c$</td>
<td>$z_0/h$</td>
<td>0.13</td>
<td>0.12</td>
<td>0.052</td>
</tr>
<tr>
<td></td>
<td>$u_{c1}^e/U_1$</td>
<td>0.0957</td>
<td>0.0914</td>
<td>0.0754</td>
</tr>
</tbody>
</table>

RS and IS denote the roughness sub-layer and inertial sub-layer, respectively.
becomes simply

$$0 = -\rho \frac{\partial \langle u'w' \rangle}{\partial z},$$

which yields the usual result that $\overline{u'w'} = \text{constant}$.

For practical flow over a rough wall in a wind tunnel with a fixed roof, the cross-sectional area of free stream flow decreases as the boundary-layer thickness increases with increasing fetch and the pressure gradient along the tunnel is thus not zero. The dispersive stress and drag terms remain negligible in the region above the top of the canopy, so Eq. (9) becomes

$$\rho \left( \overline{U} \frac{\partial \overline{U}}{\partial x} + \overline{W} \frac{\partial \overline{U}}{\partial z} \right) + \frac{dP}{dx} + \rho \frac{\partial \langle u'^2 \rangle}{\partial x} = -\rho \frac{\partial \langle \overline{u'w'} \rangle}{\partial z}. \quad (12)$$

Each term was estimated from measurements over a staggered, uniform 10 mm cube array with packing density of 25% at similar conditions to those of the present study. This showed that the first and second terms on the left-hand side were negligible near the surface (though significant near the boundary-layer edge), even though accurate estimation from experimental data is not possible due to the very small values of $\partial \overline{U}/\partial x$ and $\overline{W}$. The axial pressure gradient, determined by measuring the free stream velocity at different fetches over the roughness, was found to be the major contributor on the left-hand side, and together with other terms roughly balances the estimated shear stress gradient near the wall. This is probably the reason why the shear stress is not constant with height in many wind tunnel measurements.

In Section 4.1, the definition of the inertial sub-layer was arbitrarily chosen by a 5% variation in the spatially averaged Reynolds shear stress; here we examine the effects of alternative definitions within the log-law region on the surface parameterisation, while using $u^*(p)$ as the friction velocity. The results are listed in Table 2. It is found that the roughness length does not vary significantly whether or not the roughness sub-layer is included in the analysis, providing that the velocity profiles in the roughness sub-layer are spatially averaged. Furthermore, the roughness length is not sensitive to whether the inertial sub-layer is defined by a 5%, 10% or even (in the case of the C20S-6.25% surface) a 20% variation in the spatially averaged shear stress profiles.

Use of the height of the centre of drag $H_c$ to define the zero-plane displacement (following Jackson’s theory, Kastner-Klein and Rotach (2004)) in the log-law analysis leads to different values for $z_0$ in each case. The results of this procedure are also listed in Table 2. For the 6.25% roughness, the values of $z_0$ differ from those obtained by alternative methods by less than the overall uncertainties and the fitting is visually good, as shown in Fig. 12. However, for the 25% surfaces, which are also displayed in Fig. 12, it is not possible to obtain a reasonably good fit without changing $u^*(p)$. This suggests that although Jackson’s theory may work well at low density, it breaks down when the flow begins to skim the roughness, at least for the cases studied here.

As mentioned above, in wind tunnel simulations the friction velocity is often derived from vertical profiles of the shear stress in the inertial sub-layer, which is here denoted as $u^*(m)$ to distinguish it from $u^*(p)$. $u^*(m)$, derived from the inertial sub-layer with 5% variation in shear stress, was also used as the friction velocity to fit the same log-law
regions for each surface, as illustrated in Fig. 12. The deduced parameters are also included Table 2. Compared with the results using $u^e(p)$ as the friction velocity, the roughness length derived with $u^e(m)$ is consistently and significantly lower than that with $u^e(p)$ for the three surfaces analysed here.

To assess errors caused by using the method adopted by Macdonald et al. (1998), $d$ was prescribed as the height of the centre of drag $H_c$, and the friction velocity and roughness length were determined by curve fitting the data in the log-law region (here the friction velocity is denoted by $u^e_c$ to distinguish from others). The results are listed in the final row of Table 2. It is clear that the roughness length can be three times larger than that determined by using $u^e(p)$, and the curve fitted friction velocities are much larger than $u^e(m)$ (e.g. by 50% for surfaces with packing density of 25%), which may partially explain the differences observed by Macdonald et al. (2000) as discussed in the next paragraph. This also raises the question of how reliable the geometrical methods might be if their parameterisation was based on inappropriate wind tunnel data (e.g. data at too short a fetch).

The results are compared with predictions from geometrical methods from the literature, such as those of Macdonald et al. (1998) and Kastner-Klein and Rotach (2004), in Fig. 13; both correlations have the correct logical behaviour of the roughness length with variations in area density. The correlation by Macdonald et al. (1998) was derived from basic theoretical principles with some simple assumptions; its constants were empirically determined by fitting to experimental data from Hall et al. (1996), and the fitting constant in the expression for the roughness length is different for different roughness patterns, depending on whether an array is staggered or not. Here, it is emphasised that Hall et al. (1996) took extensive measurements with pulsed wire anemometry over a series of cube arrays in a wind tunnel at a fetch no more than $22h$. Because no direct measurements of the shear stress were made, the three quantities ($z_0$, $d$ and $u^e$) had to be found by log-law fitting the mean wind speed profiles. Compared with the subsequent roughness length data of Macdonald et al. (2000), where $d$ was estimated by the correlation in Macdonald et al. (1998), the scatter is significant. In the experiments of both Hall et al. (1996) and Macdonald et al. (2000) the fetch was relatively short and it is possible that in both cases the flow did not have a properly developed equilibrium log-law layer. In addition, Macdonald et al. (2000) noticed that the effective $u^e$ derived by fitting the mean wind profile to the log-law with three unknowns of $u^e$, $d$ and $z_0$ was consistently 75% larger than $u^e(m)$ determined directly from their shear stress measurements and they were greatly puzzled by the poor agreement between the two methods.

Comparison of our data with the prediction by Macdonald et al. (1998) shows that the $z_0$ values are lower and $d$ larger than predicted; this may be because the $u^e(p)$ in our experiment was independently determined and our fetch (150$h$) is significantly larger than theirs ($22h$). Also, because the velocity profile was spatially averaged over a modular unit, the roughness length and zero-plane displacement from our experiments should be more representative of the given surfaces. For comparison purposes, the correlations by Kastner-Klein and Rotach (2004) are also included in Fig. 13. Our data seem to be closer to the latter predictions than those from Macdonald et al. (1998). However, considering how the correlations were developed, it seems that if the constants were to be obtained by fitting better quality wind tunnel data at greater fetch, the correlation by Macdonald et al. (1998) would have the potential to perform better.
5. Conclusions

The relationships between surface drag and the flow field properties in the inertial sub-layer over regular roughness arrays have been investigated in detail. The drag on both a
single roughness element and a group of elements was measured directly. Vertical profiles of mean flow and Reynolds stresses were obtained over the module area associated with a single roughness element and then spatially averaged. The analysis then proceeded with the spatially averaged profiles and the surface drag measurements. The experiments were performed over extensive \((x/h = 150)\) idealised urban surfaces of different packing densities at locations where the ratio \(\delta/h\) took the value of \(6\sim7\); though rather low, this value is not untypical of wind tunnel studies of urban boundary layers.

The log-law region could be extended to the roughness sub-layer provided the velocity profiles were spatially averaged. Within the log-law region, the shear stress was not strictly constant with height, though the value deduced by downward extrapolation to the zero-plane level was consistent with values obtained by surface drag measurement. The roughness length determined by log-law fitting did not change significantly whether or not the roughness sub-layer was included in the analysis.

The surface shear stress deduced from form drag measurements was consistently larger than the Reynolds shear stress in the log-law region. Therefore, using a friction velocity derived from Reynolds shear stress data in the log-law fitting process can clearly lead to significant errors in the deduced roughness length. The results also showed that the height of the centre of drag, calculated from the pressure forces acting on a roughness element, was not a reliable predictor of the zero-plane displacement.

The accuracy of geometrical methods of determining the aerodynamic properties of rough surfaces was assessed. This showed that the performance of such methods could be seriously jeopardised if their parameters were derived from inappropriate wind tunnel data (e.g. obtained at too short a fetch).

Finally, we have shown that at sufficiently low area coverage, the roughness sub-layer can extend over a significant portion of the boundary-layer depth, depending on wind direction. Whether the resulting extensive spatial inhomogeneity would persist at larger fetches (i.e. at greater \(\delta/h\)) remains an open question. It is hoped that this study can stimulate more research on this important but not well-understood topic.

Acknowledgements

We are grateful for the technical support provided by Mr. A. Wells and T. Lawton; without their assistance none of these experiments would have been possible. HC was funded by EPSRC under the Grant No. GR/R78084/01(P) and PH is funded by NERC under the UWERN programme. That financial support is greatly appreciated.

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